UDC 533.6.011;534.222.2

ON THE PROBLEM OF SHOCK WAVE FORMATION WITHIN A LOCAL SUPERSONIC ZONE

PMM Vol. 43, No. 6, 1979, pp. 1117-1118

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(Received January 10, 1979)

A particular exact solution of the transonic equation is considered as an extension of Frankl's solution [1] for the flow at some distance from the profile. The derived generalization defines the flow over some half-body of a gas stream at a velocity that is subsonic at infinity.

The characteristics of a set are reflected from the sonic line and, being compression waves after reflection, they begin to intersect at some point K inside a local supersonic zone forming the envelope (boundary line) with a cusp at point K. The distance of point K from the sonic line proves to be fairly small, and its magnitude may be used for evaluating the accuracy of numerical methods.

The conclusion about shock wave formation not on the sonic line but inside a local supersonic zone was recently arrived at by a number of authors with the use of numerical methods [2 — 4].

Let us consider the approximate system of transonic equations

$$uu_x = v_y, \quad u_y = v_x \tag{1}$$

where u and v are normalized dimensionless perturbation velocities of a uniform stream, and x, y are Cartesian coordinates.

An exact solution of Eq. (1) is

$$u = \frac{C_1(1 - C_2s^3)}{2s^2} + \frac{C_1^2t^2}{4} \tag{2}$$

$$v = \frac{C_1^2(2 + C_2s^3)}{4s^2}t - \frac{C_1^3t^3}{12}$$

$$x = \frac{1 + 2C_2s^3}{2s^2} - \frac{C_1t^2}{4}, \quad y = t$$

where s and t are parameters, and C_1, C_2 are arbitrary constants. This solution defines the flow in plane Laval nozzles with a local supersonic zone at its walls. Such flow was analyzed in [5]. Note that when $C_2 = 0$, a self-similar nozzle flow with linear distribution of the longitudinal velocity $(u = C_1x)$ on the axis $y = 0$ is obtained from (2).

It can be shown that besides the solution (x, y, u, v) defined by formulas (2) there exists a multiplicity of solutions (x_i, y_i, u, v) of Eq. (1), with

$$x_i = \frac{\partial^{(i)}x}{\partial v^i}, \quad y_i = \frac{\partial^{(i)}y}{\partial v^i} \tag{3}$$

generated by differentiation operators

$$\frac{\partial}{\partial v} = C_1^{-2}k^{-1}\left[4s^5tC_1\frac{\partial}{\partial s} + 4s^2(2+C_2s^3)\frac{\partial}{\partial t}\right] \tag{4}$$
$$k = (2+C_2s^3)^2 - 6C_1s^2t^2$$

The proof is implied by the form of Eqs. (1) written in the hodograph plane $uy_v = x_u, y_u = x_v$, and by the possibility of differentiating x and y with respect to the variable v for obtaining new solutions.

For $i = 2$ we have

$$x_2 = \frac{8s^4\delta_1}{C_1^3k^3}, \quad y_2 = \frac{16s^6t\delta_2}{C_1^3k^3} \tag{5}$$
$$\delta_1 = (C_2s^3+2)^3(C_2s^3-4) - C_1s^2t^2(C_2s^3+2)(C_2^2s^6 - 32\,C_2s^3 + 40) - 18\,C_1^2C_2s^7t^4$$
$$\delta_2 = (C_2s^3+2)^2(16 - C_2s^3) - 18\,C_1C_2s^5t^2$$

The respective potential φ is of the form

$$\varphi = ux_2 + vy_2 - \frac{4s^2(2+C_2s^3)}{C_1^2k} \tag{6}$$
$$(u, v) = \mathrm{grad}_{x,y}\,\varphi$$

When $C_2 = 0$ and $C_1 = C_{11} = -2^{-8}3^{-3}5^5$, x_2, y_2 together with u, v from (2) define the self-similar solution [1] with the limit characteristic defined by $x_2y_2^{-4/5} = 1$. For fixed C_1 and $C_2 \neq 0$, $t = 0, C_2s^3 = -2$ with the characteristic subsonic velocity

$$u_0 = {}^3/_2\, C_1(-C_2/2)^{2/3}$$

correspond to infinity in the x_2y_2-plane.

Let us consider the branch of solution (5), (6) which becomes a part of solution [1] which defines the flow ahead of the shock wave, as $C_2 \to 0$. For x_2, and $y_2 \to 0$ both flows ($C_2 = 0$, $C_2 \neq 0$) asymptotically merge. The sonic line begins in the form of the generalized parabola

$$x_2y_2^{-4/5} = 5^{-1}2^{4/5}3^{8/5}\,(C_1 = 2\,C_{11},\ C_2 = -0.09496)$$

but, then, at $y_2 \approx 22$ turns abruptly downward creating a local supersonic zone. The condition $v = 0$ is satisfied only at $y_2 = 0, x_2 < 0$, while at $y_2 = 0, x_2 > 0$ $v \neq 0$. The top of the local supersonic zone is shown in Fig. 1 at the spot where a three-sheet fold is formed. The solid line represents the sonic line and the dash line corresponds to the limit line $I = 0$, where I is the Jacobian of transormation $D(x_2, y_2)/D(s, t)$.

The limit line consists of two branches one of which has a common vertical tangent with the sonic line. The continuation of this branch downward brings it to the coordinate origin, and the second branch asymptotically approaches the axis $y_2 = 0$ with $x_2 > 0$. The cusp K of the limit line lies inside the local supersonic zone, its distance from the sonic line along the straight line $y = y_K$ is $\Delta x = 0.01$ which is small as compared to the over-all zone height $\Delta h \approx 22$.

If the solution has to have any physical meaning, it is necessary to construct the shock wave which would eliminate the ambiguity of solution in the physical plane. In the considered class of solutions it is impossible to satisfy along a single curve the

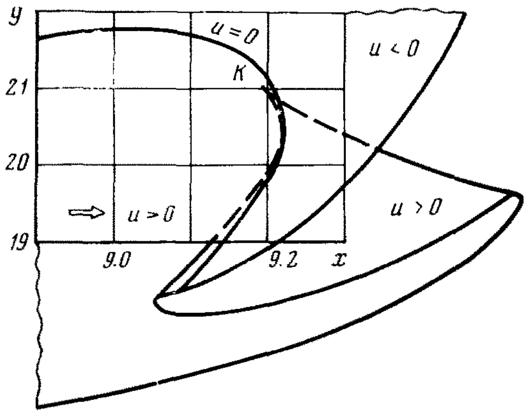

Fig. 1

two conditions of continuity of the potential and of the polar line.

However the formation of a fold within a local supersonic zone, as shown by the above example, supports the argument in favor of the possibility of shock wave origination inside that zone. Thus, if one considers the streamline passing slightly below point K as a solid wall, then, taking into account the shock wave weakness, one must expect its appearance as close to point K, as desired.

The authors thank S. V. Fal'kovich for his interest in this work.

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Translated by J. J. D.